Bounding Solitaire Scrabble Scores

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Abstract

Using any dictionary and allowing for all future words, what are the highest and lowest scores possible using all 100 Scrabble tiles? A new highest score of 5898 is conjectured, while the previous record low of 216 was not improved.

Since we cannot exclude any combination of letters from being added to some dictionary, we will only consider the point value of each tile: 10, 10, 8, 8, ..., 1, and 0, 0 for the two blanks. Some of these values will be counted multiple times towards a player's score due to bonus squares and re-use in connecting words. We can construct a frequency diagram of these counts which may describe a vast number of hypothetical games. An example is given in Figure 1.

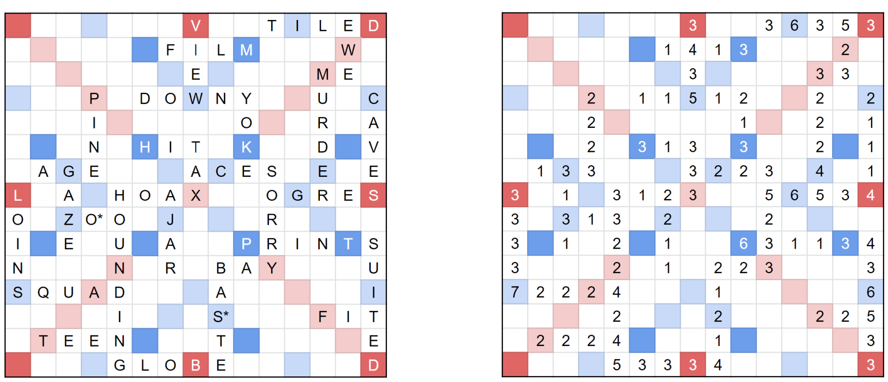


Figure 1: A final game board alongside a possible frequency diagram.

The theoretical maximum score for any 100-tile frequency diagram is the dot product of the frequencies with the tile values, both in descending order. The minimum score is found by reversing one of these vectors.

Firstly, to maximise total score we should try to front-load our frequency vector, ensuring the 10- and 8-point tiles are counted many times. The highest possible frequency is 71. This is achieved by playing on a double-letter square with a triple-triple-word multiplier (54) and a single neighbour on the other axis (+2) then extending to the other side of the board by one letter at a time (+15 with double-word squares). This can be done twice on the same board. Given these, the next two highest frequencies are 56, given by the opposing double-letter squares in a second triple-triple-word play. After maximising the top four frequencies we get to tile values ≤ 5, which makes bingoing often more valuable than incremental extension.

This front-loading technique can be observed in the frequency diagram of S. Root and C. F. Brown's 99-tile, 5876-point game (Figure 2) [1]:

30	7												7	30
30	6												6	30
29	6											6	8	31
56	7	3	5	6	7	8	10	10	11	12	14	20	21	71
30	6											7	7	31
30	8											7	9	31
29	6											9	7	30
30	9	5	7	7	8	14	14	14	14	14	16	20	17	38
30	6											9	7	31
30	15											7	16	31
29	5											7	6	30
56	6	3	5	6	7	8	10	10	11	12	14	21	20	71
28	5											6	7	30
28	5												5	28
27														27

Figure 2: Frequency diagram of a theoretical, high-scoring game.

Features to note in this diagram are the incremental stacks on the two highest-frequency squares, the two-bingo set-up in each of columns B and N for the optimal triple-triple-word plays, and the 8G opening bingo (for a total of 8 bingos). The tiles from C8 to F8 and most of column M are less important and just serve to best use up the remaining tiles.

Using the same base model for a 100-tile game, there are several approaches one might take. Note that we could now start with an 8H bingo and the extra tile would appear on N15 after two bingos in column N (Figure 3-a). This gives a greater frequency for the fifth highest tile value with minimal loss elsewhere. We can also squeeze in a ninth bingo using some of the extraneous tiles mentioned above, but this doesn't increase the score (Figure 3-b). In fact, it seems the best score is achieved by the naive approach of adding the N15 tile just before the first triple-triple-word play in Root's game. This maximum is show in Figure 3-c.

30	7												7	30
30	6												6	30
29	6											6	8	31
56	7	3	5	6	7	8	10	10	11	12	14	20	21	71
30	6											7	7	31
30	8											7	9	31
29	6											9	7	30
30	9	5	7	7	8	9	14	14	14	14	16	20	19	39
30	6											9	7	31
30	15											7	9	31
29	5											7	6	30
56	6	3	5	6	7	8	10	10	11	12	14	21	20	71
28	5											6	7	30
28	5												5	28
27													7	30

(a) 5882 with 8H start

30	7												7	30
30	6												6	30
29	6											6	8	31
56	7	3	5	6	7	8	10	10	11	12	18	19	21	71
30	6										5	6	8	32
30	8										5	6	10	32
29	6										6	9	8	31
30	8					6	6	6	6	6	13	10	9	30
30	6										5	9	8	32
30	15										5	6	17	32
29	5										5	4	7	31
56	6	3	5	6	7	8	10	10	11	12	18	16	20	71
28	5												5	28
28	5												5	28
27														27

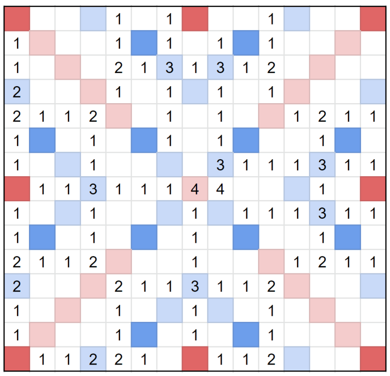
(b) 5840 with 9 bingos

30	7												8	30
30	6												7	30
29	6											6	9	31
56	7	3	5	6	7	8	10	10	11	12	14	20	22	71
30	6											7	8	31
30	8											7	10	31
29	6											9	8	30
30	9	5	7	7	8	14	14	14	14	14	16	20	18	38
30	6											9	8	31
30	15											7	17	31
29	5											7	7	30
56	6	3	5	6	7	8	10	10	11	12	14	21	21	71
28	5											6	8	30
28	5												6	28
27													4	30

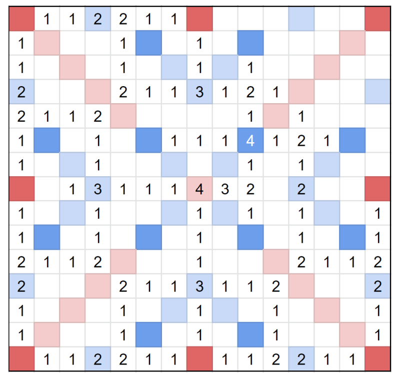
(c) Maximum of 5898

Figure 3: Theoretical high-scoring frequency diagrams.

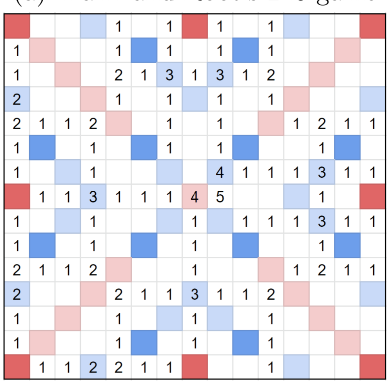
While this score is unlikely to ever be attained with legal words, a lowest-scoring game may well be possible. A. Frank and S. Root came up with a theoretical 216-point, 100-tile game in 1981 (Figure 4-a) [2] and while there are errors in their attempt to realise this score, it seems to be the lower limit. Their score is within 50 points of the sum of all tile values: 187. A lowest-scoring game will therefore never contain a play of more than 6 tiles. When searching for low-scoring games it is tempting to assume other heuristics, but not all are necessarily sound. For example, we can construct a 217-point diagram that crosses a triple-letter square (Figure 4-b). There is even some wiggle room with 216 points: we can reach the same score while placing a non-blank tile on the first play (Figure 4-c). It is also possible to make a 215-point diagram with one disconnected tile (Figure 4-d) and Frank and Root do not play optimal word lengths in their 216 game.



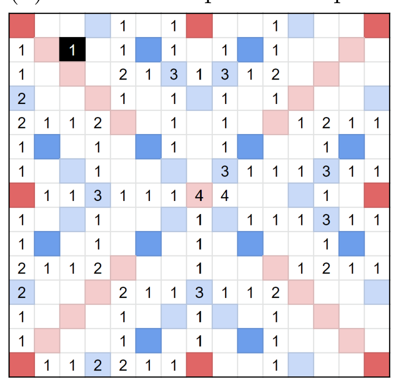
(a) Frank and Root's 216 game



(b) 217 with a triple-letter square



(c) 216 with a 2-point first turn



(d) 215 with one loose tile

Figure 4: Low-scoring frequency diagrams.

While we cannot improve on the previous low-score record, we can bring the number of plays required down to 22. One such frequency diagram and an example board using only well-known words are shown in Figure 5.

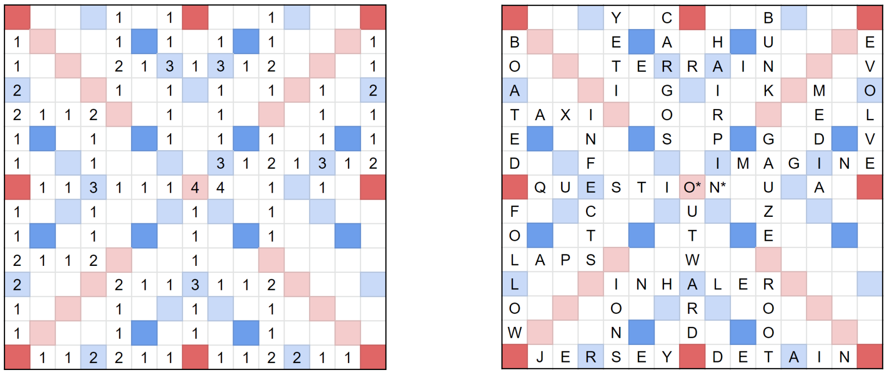


Figure 5: Frequencies and example board for a 22-turn 216-point game.

The example game can be played out as follows:

1. on 8H +0
2. QUESTI(on) 8B +16
3. INF(E)CTS D5 +12
4. TAX(I) 5A +11
5. BOA(T)ED A2 +10
6. LAP(S) 11A +6
7. FO(L)LOW A9 +13
8. (o)UTWARD H8 +11
9. INH(A)LER 12E +10
10. (I)ONS E12 +4
11. JER(S)EY 15B +17
12. (R)OOT K12 +4
13. DE(T)AIN 15I +8
14. HAIRPI(n) I2 +13
15. TERR(A)IN 3E +8
16. YE(T)I E1 +7
17. CA(R)GOS G1 +9
18. BU(N)K K1 +10
19. (I)MAGINE 7I +11
20. G(A)UZE K6 +15
21. MED(I)A M4 +8
22. EVOLV(E) O2 +13

References

[1] Root, Stephen C., *No-Dictionary Scrabble Revisited*.
Word Ways: Vol. 16 : Iss. 1 (1983), Article 13.
Retrieved from: https://digitalcommons.butler.edu/wordways/vol16/iss1/13

[2] Frank, Alan, *Low-Scoring Scrabble Games*.
Word Ways: Vol. 14 : Iss. 3 (1981), Article 17.
Retrieved from: https://digitalcommons.butler.edu/wordways/vol14/iss3/17